INTRODUCTION TO FORECASTING

INTRODUCTION:

What is a forecast? Why do managers need to forecast?

A forecast is an estimate of uncertain future events (literally, to "cast forward" by extrapolating from past and current data).

Forecasts are used to improve decision-making and planning. Even though forecasts are almost always in error, it is better to have the limited information provided by a forecast than to make decisions in total ignorance about the future.

EXAMPLE: Demand (sales) forecasting -- Firms must anticipate future demand to best plan how to satisfy it through on-hand inventory or available capacity. The production or procurement lead time often requires production and ordering decisions to be made before demand occurs.

FORECASTING DECISION-MAKING ENVIRONMENTS:

<table>
<thead>
<tr>
<th>Short term (0-6 months)</th>
<th>Long term (2+ years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational decisions</td>
<td>Strategic decisions</td>
</tr>
<tr>
<td>Frequently made</td>
<td>Infrequently made</td>
</tr>
<tr>
<td>Low level of responsibility</td>
<td>Top management level</td>
</tr>
<tr>
<td>Individual items</td>
<td>Product line</td>
</tr>
</tbody>
</table>

/\    /\    /\
weekly demand    annual (monthly) demand    10-year demand
(inventory control) (production planning) (facility planning)

QUALITATIVE VS. QUANTITATIVE METHODS

Qualitative forecasting techniques (soliciting opinions):

- subjective, based on the opinion and judgment of consumers, experts, managers, salespersons
- appropriate when past data are unavailable (new product) or when past data are not reliable predictors of the future
- usually applied to intermediate -- long range decisions

Quantitative forecasting techniques:

- explicit mathematical models are used to estimate future demand as a function of past data
- appropriate when past data are available and also are reliable predictors of the future
- usually applied to short -- intermediate range decisions
QUALITATIVE FORECASTING METHODS

1. Informed opinion and judgment:
   - subjective opinion of one or more individuals
   - accuracy of the forecast depends on the individuals
   - EXAMPLE: ("grass roots") collection and aggregation of individual sales forecasts to obtain overall sales forecast by product or region

2. Delphi method: an iterative technique for obtaining a consensus forecast from a group of experts, without the problems inherent in group decision-making (the "bandwagon" effect, influential individuals). The procedure works as follows: first, give a set of questions to each expert, who provides answers (forecasts) independently from the other experts. The responses are collected and numeric responses are statistically summarized. If a consensus was not obtained, return the summarized responses to the experts, along with any comments made by the experts (anonymously), and have them revise their forecasts based on this data. Repeat until either a consensus is reached (the answers converge) or else a "stalemate" occurs (no convergence can be obtained).
   - EX: long range forecasting of technological advances

3. Market research: Questionnaires and interviews are used to solicit the of potential customers, current users, and others. One potential problem is that stated intentions (expectations) do not always translate into behavior.
   - EX: voter preferences, new car buyers

4. Historical Life-Cycle Analogy: Demand for a new product can be forecast by anticipating an S-shaped growth curve similar (analogous) to the S-curve experienced with related products

<table>
<thead>
<tr>
<th>PRODUCT LIFE CYCLE CURVE</th>
</tr>
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<tbody>
<tr>
<td>Sales per time period (units)</td>
</tr>
<tr>
<td>growth</td>
</tr>
</tbody>
</table>
QUANTITATIVE FORECASTING TECHNIQUES

TIME SERIES ANALYSIS:
- Assumes that patterns in demand are due to time
- Projects past data patterns into the future (extrapolates from historical demand)

Time Series Decomposition: decompose (break down) the pattern into level, trend, seasonal, cyclical, and random components.
- the random component is, by definition, unpredictable
- the cyclical component is due to long term (several years) business/economic cycles and thus is very difficult to identify
- time series methods usually try to identify the seasonal (a cycle that repeats yearly), trend, and level components

Time Series Methods:

1. Last period demand (often called the "naive" forecast)

\[ F_{t+1} = A_t \]

2. Arithmetic Average: average of all past demand, to "average out" or "smooth out" the random fluctuations

\[ F_{t+1} = \frac{\sum_{i=1}^{t} A_i}{t} = (A_1 + A_2 + \ldots + A_t) / t \]

3. Simple Moving Average (N-Period): average of the N most recent demands, to "smooth out" the random fluctuations -- the average "moves" to include the most current data (in case demand really isn't flat)

\[ F_{t+1} = \frac{\sum_{i=t+1-N}^{t} A_i}{N} \]

Choosing the value of N involves a tradeoff between stability (the ability to maintain consistency and not be influenced by random fluctuations) and responsiveness (the ability to adjust quickly to true changes in the demand level):

- large N means the average is stable but less responsive to changes in demand
- small N means the average is less stable but will respond more quickly to changes in demand
4. Weighted Moving Average (N-Period):
- The weights are chosen so that they sum to 1
- If all weights are equal \( (W_i = 1/N \text{ for all } i) \), the weighted moving average is equivalent to the simple moving average.

\[
F_{t+1} = \sum_{i=1}^{N} W_i A_{t-i+1}
\]

- Advantage: can vary weights to emphasize more recent data
- Disadvantage: to change responsiveness, must change weights individually; requires recording or storing N weights and N past demands

5. Simple Exponential Smoothing: a simple way of calculating a weighted moving average forecast with exponentially-declining weights; only the previous forecast, most recent demand, and the value of a smoothing constant are needed to calculate the new forecast. Another way of writing the equation clearly shows that the new forecast is equal to the old forecast plus an adjustment, where the adjustment is calculated as the smoothing constant times the previous forecast error:

\[
F_{t+1} = \alpha A_t + (1-\alpha) F_t, \text{ where } 0 \leq \alpha \leq 1
\]

The value of a smoothing constant, \( \alpha \), determines how much of an adjustment or correction will be made in response to the most recent demand. Large \( \alpha \) means a larger adjustment (more weight given to recent data, giving less stable and more responsive forecasts), while small \( \alpha \) means a smaller adjustment (more weight given to older data, giving more stable and less responsive forecasts).

An equivalence (this only means the forecasts will be similar, not identical) between an N-period simple moving average and an exponentially-smoothed average is obtained by setting \( \alpha = 2/(N+1) \).

The exponential smoothing procedure yields a weighted moving average with exponentially-declining weights and an "infinite" number of terms (all past demand data back to time \( t=1 \) is given at least some weight). The weights given to the individual demands can be calculated using the following formula (\( W_1 \) is the weight given to the current period's demand, \( W_2 \) is the weight given to the next most recent demand, and so on):

\[
W_k = \alpha(1-\alpha)^{k-1}
\]
Example: $\alpha = .1$, $F_1 = 100$, $A_1 = 105$

What happens with the extreme values of $\alpha$?

With simple exponential smoothing, the forecast for any period in the future (for example, the pth period beyond the current period) is the same value:

$$F_{t+p} = F_{t+1}, \text{ for } p > 1.$$  
(It is likely that this forecast will decline in accuracy the further into the future that it is projected.)

6. **Calculating Multiplicative Seasonal Indexes:**

1. collect monthly (quarterly) demand data for several past years

2. for each month (quarter) of past data, calculate the ratio of demand to a 4-quarter (12-month) moving average

3. average the ratios for several years of a given quarter (month) to get the seasonal index for that quarter (month)

Example: Demand (000's of units)

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Q2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Q3</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Q4</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
EVALUATING FORECAST QUALITY

Forecast error, or a related performance measure, can be used to select a forecasting method that has the smallest forecast error, and to monitor the performance of a forecasting method in use.

Error is the difference between actual demand and forecast demand: error = \( A_k - F_k \). The cumulative error is often called the running sum of forecast errors (RSFE).

The mean forecast error (MFE), often called average cumulative error or bias, measures the tendency of a forecasting model to consistently overforecast or underforecast.

\[
RSFE_t = \sum_{k=l}^{t} (A_k - F_k), \quad MFE_t = BIAS = \frac{\sum_{k=l}^{t} (A_k - F_k)}{t}
\]

If bias > 0, forecasts consistently are too low, and if bias < 0, forecasts consistently are too high. Ideally, bias will be close to zero. The primary drawback to using bias alone to evaluate forecast quality is that positive and negative errors tend to cancel. A related performance measure that does not have this problem is the mean absolute deviation (MAD):

\[
MAD_t = \frac{\sum_{k=l}^{t} |A_k - F_k|}{t}
\]

The MAD is related to the standard deviation \( \sigma \) in that for normally-distributed forecast errors, \( \sigma \approx 1.25 \text{MAD} \). A third forecasting performance measure is mean squared error (MSE):

\[
MSE_t = \frac{\sum_{k=l}^{t} (A_k - F_k)^2}{t}
\]

MSE has the same advantage over bias that MAD has, namely, positive and negative errors do not cancel each other out. The difference between MSE and MAD is that MSE penalizes large errors much more than MAD does.

To select a forecasting method from several potential models, one approach is to take a series of actual historical data (demand) and apply each model to the data. The method that yields the smallest MAD or MSE and has bias close to zero usually is the preferred method. To monitor the performance of a forecasting model in use, a tracking signal is often used:

\[
TS_t = \frac{RSFE_t}{MAD_t}
\]

Ideally, the tracking signal will be close to zero, but values within a specified range (for example, \(-4 \leq TS \leq 4\)) are considered acceptable. If the tracking signal falls outside of the acceptable range (this may occur if the underlying demand pattern has changed sharply), stop and reset the forecast. Some researchers feel that simple exponential smoothing with tracking signal control is better than trend-adjusted exponential smoothing.
CAUSAL FORECASTING

Causal forecasting is appropriate when there is a "cause and effect" relationship between one or more independent variables (the "cause") and a dependent variable (the "effect") such as demand or some other variable that is being forecast. Causal models have the potential to predict turning points in the demand function, something that time series models can not do. (Why?)

The general approach to causal forecasting is:

1. collect historical data
2. develop and validate the model
3. use the model to forecast

Multiple regression analysis is one approach used to develop a causal forecasting model. It is important to note that regression implies dependence and not necessarily causation, however, causation does not have to be proven for a causal forecasting model to be used effectively. The general form for a multiple linear regression equation is:

\[ Y_c = a + b_1X_1 + b_2X_2 + \ldots + b_kX_k \]

\( Y_c \) = calculated (predicted) value of the dependent variable

\( a \) = intercept (constant term)

\( X_j \) = jth independent (predictor) variable

\( b_j \) = coefficient associated with the jth independent variable

A computer (or programmable calculator) is used for calculating the intercept (a) and slope (b) coefficients. The point (single value) forecast made with this model is the value of \( Y_c \) after current values for the \( X_j \)'s have been inserted.

Before a causal forecasting model is used it must be validated. This means to check whether the model contains only variables that significantly help make an accurate forecast. Following the "principle of parsimony", the simplest model (the one having the fewest variables) that gives good results should be selected. Larger models, with more variables, will have a smaller bias component (good) but also result in larger forecast variance (bad). Some factors that help in validating a causal model include:

1. R-SQUARE \((r^2, \text{the Coefficient of Determination})\) measures the percentage of variation in the data that is explained by the model. R-SQUARE can take any value between zero and one. Ideally, R-SQUARE will be close to one. Although a large R-SQUARE value is desirable, the model with the largest R-SQUARE may not be the best model. Each variable included in the model will contribute to R-SQUARE, so the model that contains all variables being considered will have the largest R-SQUARE of any model. However, if some variables do not significantly contribute to the model, it is better to drop them from the model and select a model with a
slightly smaller R-SQUARE.

2. **ADJUSTED R-SQUARE** is a variation of R-SQUARE that penalizes for overfitting the model (including too many variables), and therefore is useful for getting a feel for how many variables should be included.

3. To test the significance of each independent variable, either a t-test or an F-test (or both) should be performed. For our purposes, the t-statistic will be sufficient evidence of the significance (or lack thereof) of a variable. The t-test for a given variable uses the null hypothesis that the coefficient of that variable is equal to zero.

A large t-value (small PROB value) indicates that the null hypothesis should be rejected, and thus the $b_i$ coefficient is likely to be non-zero and the variable should be included in the model. Conversely, a small t-value (large PROB value) implies that the null hypothesis should be accepted, and therefore the $b_i$ coefficient is not significantly different from zero. In this latter case, the variable should not be included in the model.

A general guideline for selecting and validating a causal forecasting model is to (1) think of all variables that may help predict the item being forecast and (2) collect historical data for several observations, where each observation contains the value of the item being forecast (for example, sales) as well as the values of the predictor variables (for example, the prime interest rate) at the same point in time. Next, the factors described above (R-SQUARE, ADJUSTED R-SQUARE, and the t-tests) can be used to (3) examine each model in detail and determine which variables should remain in the final model.
An Example of Causal Forecasting Using Multiple Regression Analysis

The manager of a real estate firm in a large metropolitan area wants to establish a model for forecasting the market value of residential property. She believes that such a model would enable her firm to decrease the length of time that a client's house remains on the market, since the asking price would be more in line with true market values.

(1) The manager thinks that the market value of a house may be influenced by four factors: the size of the house, the distance that the house is from the business district, the condition of the house, and the size of the lot.

(2) A random sample of 10 houses sold within the past two months generated the following data:

<table>
<thead>
<tr>
<th>[MV] Market Value (selling price)</th>
<th>[SQFT] Size of House (in square feet)</th>
<th>[DIST] Distance from Business Dist. of House (in miles)</th>
<th>[COND] Condition (0 - 10)</th>
<th>[LOTSIZE] Size of Lot (in acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 50,000</td>
<td>1,200</td>
<td>6</td>
<td>7</td>
<td>0.5</td>
</tr>
<tr>
<td>$ 90,000</td>
<td>2,500</td>
<td>8</td>
<td>9</td>
<td>0.5</td>
</tr>
<tr>
<td>$ 72,000</td>
<td>3,000</td>
<td>5</td>
<td>5</td>
<td>2.0</td>
</tr>
<tr>
<td>$ 42,000</td>
<td>1,000</td>
<td>3</td>
<td>8</td>
<td>0.5</td>
</tr>
<tr>
<td>$ 120,000</td>
<td>3,500</td>
<td>25</td>
<td>10</td>
<td>10.0</td>
</tr>
<tr>
<td>$ 75,000</td>
<td>2,000</td>
<td>10</td>
<td>7</td>
<td>5.0</td>
</tr>
<tr>
<td>$ 35,000</td>
<td>1,000</td>
<td>2</td>
<td>6</td>
<td>1.0</td>
</tr>
<tr>
<td>$ 75,000</td>
<td>2,000</td>
<td>9</td>
<td>9</td>
<td>1.0</td>
</tr>
<tr>
<td>$ 60,000</td>
<td>1,500</td>
<td>8</td>
<td>8</td>
<td>5.0</td>
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<tr>
<td>$ 100,000</td>
<td>3,000</td>
<td>5</td>
<td>9</td>
<td>2.0</td>
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