The Ballistic Pendulum

September 26, 2006
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Purpose: By doing this experiment, we will be able to calculate the initial velocity of a projectile by using a ball in a ballistic pendulum. Conservation of momentum and energy will apply as well as error analysis.

Theory: The idea behind this lab is to see how velocity and position are related for projectiles. We should discover that energy and momentum are conserved in any situation. Hopefully, we can use these ideas to be able to predict where a projectile will land with a given initial velocity.

Procedure:

The ballistic pendulum and ball being launched will be weighed separately, and their masses are recorded below.

\[ m_{\text{ball}} = 16.9 \, \text{g} \quad m_{\text{pendulum}} = 241.5 \, \text{g} \]

We will then place the ball inside the pendulum and mark its center of mass. After reattaching the pendulum on the system, we will measure its height from the system’s base to the center of mass mark. This is the projectile’s initial height:

\[ h = 1.0 \, \text{m} \]

The pendulum apparatus looks somewhat like this:
we will now fire the gun and measure the Final height that the ball reaches, estimating uncertainty.

\[ \overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \]

\[ \sigma_x = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n-1}} \]

\[ \overline{x} = \frac{9\text{cm} + 8.2\text{cm} + 8.0\text{cm} + 9\text{cm} + 8.5\text{cm}}{5} = 8.8\text{cm} \]

\[ \sigma_x = \sqrt{\frac{(0.2\text{cm})^2 + (0.1\text{cm})^2 + (0.3\text{cm})^2 + (0.2\text{cm})^2 + (0.3\text{cm})^2}{4}} \]

\[ \sigma_x = 0.2\text{cm} \]  

Average \[ h_1 = 8.8\text{cm} \pm 0.2\text{cm} \]  

(1st Spring position)

We will now measure \[ h_2 \] for two stronger spring positions:

\[ \overline{x} = 10.1\text{cm} \]

\[ \sigma_x = 0.2\text{cm} \]

\[ h_2 = 10.2\text{cm} \pm 0.2\text{cm} \]  

(2nd Spring position)

\[ \overline{x} = 12.4\text{cm} \]

\[ \sigma_x = 0.2\text{cm} \]

\[ h_3 = 12.4\text{cm} \pm 0.2\text{cm} \]  

(3rd Spring position)
Using the results, we will now calculate the average height change of the ball/pendulum system:

\[ \Delta h = h_2 - h_1 \]

1st position  
2nd position  
3rd position  

<table>
<thead>
<tr>
<th>Position</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>8.8 cm</td>
<td>6 cm</td>
</tr>
<tr>
<td>2nd</td>
<td>10.7 cm</td>
<td>7 cm</td>
</tr>
<tr>
<td>3rd</td>
<td>12.4 cm</td>
<td>6 cm</td>
</tr>
</tbody>
</table>

\[ \Delta h = 2.8 \text{ cm} \]

\[ \Delta h = 3.7 \text{ cm} \]

\[ \Delta h = 6.4 \text{ cm} \]

\[ \text{Error} = 0.2 \text{ cm} \]

We will now use the law of energy conservation to calculate the average change in potential energy:

\[ E_p = m \Delta h \]

\[ (E_p = \text{Potential Energy}, \ m = \text{mass}, \ g = 9.8 \text{ m/s}^2) \]

1st position  
2nd position  
3rd position  

\[ E_p = 0.0 \text{ J} \]

\[ E_p = 0.085 \text{ J} \]

\[ E_p = 0.11 \text{ J} \]

\[ E_p = 0.2 \text{ J} \]

We will now use the law of momentum conservation to calculate the velocity of the ball before it collides with the pendulum and the velocity instantly after collision:

1st position  

\[ (M+m)g \Delta h = \frac{1}{2} (M+m)V^2 \]

\[ V = \sqrt{\frac{2g \Delta h (M+m)}{M+m}} \]

\[ V = \sqrt{\frac{2 \times 9.8 \text{ m/s}^2 \times 6 \text{ cm}}{8 \text{ cm}}} \]

\[ V = \sqrt{\frac{2 \times 9.8 \text{ m/s}^2 \times 6 \text{ cm}}{8 \text{ cm}}} \]

\[ V = 0.8 \text{ m/s} \]

\[ V = \text{Velocity instantly after collision} = 0.8 \text{ m/s} \]

\[ \frac{V}{m} = 0.068 \text{ kg} \]

\[ \frac{V}{m} = 0.25 \text{ m/s} \]

\[ \text{Velocity of ball before collision} = 0.8 \text{ m/s} \]
Conclusions: Show that the velocity before a collision is directly proportional to velocity after collision with a constant mass. This holds true to conservation of momentum. Also, a faster initial velocity is directly proportional to a change in height. This holds true to conservation of energy with a constant mass.

Now we will fire a projectile off a table along the horizontal and predict the range. It travels using our data results, we are going to use the 3rd spring position:

\[ \Delta x = V_{ox}t + \frac{1}{2}at^2 \]
\[ \Delta y = V_{oy}t + \frac{1}{2}at^2 \]
\[ t = \frac{\Delta y}{V_{oy}} \]
\[ \Delta x = 5.7 \text{ m/s} \left( 0.48 \text{ s} \right) \]
\[ \Delta y = \frac{\Delta v}{11.5 \text{ m/s}^2} \]
\[ t = \frac{\sqrt{11.5 \text{ m/s}^2}}{11.5 \text{ m/s}} \]
\[ t = 0.48 \text{ s} \]

Our projectile did not make the box.
Questions:

1. \( E_k = 0.5 \cdot (0.2 \text{ kg}) \cdot (11 \text{ m/s})^2 \) (30° position)

\( E_k = 0.2 \) \( \rightarrow \) 100% loss

At the beginning, energy is fully kinetic. At the end, energy is fully potential, so all kinetic energy is lost during the collision as it rises to max height.

2. If there were any errors in meterstick measurements, or initial velocity we calculated and had been off due to rounding results, and we did not take into account air resistance forces, little as they are.