Chapter 4

Force Tables and Vectors

4.1 Equipment List:

- Force Table Apparatus
- Three Mass Hangers
- Small Laboratory Weights
- Sprit level
- Ruler
- Protractor
- Unknown mass
- Graph Paper

4.2 Introduction:

Some physical quantities have only magnitude as, for example, volume and density. These quantities are called scalars. The student is familiar with addition, subtraction, multiplication and division of scalars. Other physical quantities such as velocity, acceleration and force need to have a direction as well as a magnitude specified. These quantities are called vectors. The algebra of vectors is more sophisticated than the algebra of scalars. In this experiment, only two aspects of the algebra are studied, that of addition and subtraction of vectors. An introduction to the multiplication of vectors will be presented course. To be even more precise, we will study in this experiment only concurrent vectors (vectors which pass through one point. The vector physical quantities, which we study here, are forces.

4.3 Physics:

4.3.1 Addition of Forces

Suppose a moving body $B$ has two forces $\vec{T}$ and $\vec{W}$ acting on it in the manner shown in figure 4.1. It may be desired to replace these forces with a single “Resultant” force $\vec{R}$
which has the same effect as the two forces \( \vec{T} \) and \( \vec{W} \). By the same effect, we mean that the body will experience the same acceleration (same magnitude and direction). The resultant force is the vector addition of \( \vec{T} \) and \( \vec{W} \). We write this as a vector equation

\[
\vec{T} + \vec{W} = \vec{R}.
\]

Of course, if \( \vec{W} \) is along (in the same direction as) \( \vec{T} \) then \( \vec{R} \) is just the scalar sum of the magnitude of \( \vec{W} \) and \( \vec{T} \), i.e.

\[
|\vec{W}| + |\vec{T}| = |\vec{R}|
\]

and the direction of \( \vec{R} \) is the same as the directions of \( \vec{W} \) and \( \vec{T} \). This can be seen in figure 4.2.

There are four methods which may be used to calculate the sum of vectors. Each of these methods may be employed in two ways:

- Make an accurate scale diagram of the forces and scale the diagram to find direction and magnitude of the sum
- Make a rough sketch so that the laws of trigonometry and geometry may be applied to find the direction and magnitude of the sum by analytical methods.

### 4.3.2 Parallelogram Rule (for two forces)

This method was tacitly used in finding \( \vec{R} \) before. To use it, construct a parallelogram using the given forces \( \vec{W} \) and \( \vec{T} \) as the two sides and making \( \vec{W}' \) and \( \vec{T}' \) parallel, respectively, see figure 4.3. (Note that \( \alpha \) is the angle between \( \vec{W} \) and \( \vec{T} \).)
4.3. PHYSICS:

The diagonal $\vec{R}$ is the vector sum. Its magnitude is found graphically by measuring $R$ to the scale used in drawing the diagram and its direction is found by measuring $\theta$ with a protractor.

For analytical calculation, the parallelogram method is no more convenient than is the triangle method, to be described next. In fact, the calculations are identical in the two cases.

4.3.3 Triangle Rule (for two forces)

This method is a special case of the more general polygon method for which the polygon is a triangle. It makes use of the fact that a vector may be moved parallel to itself with impunity. Form two sides of a triangle with $\vec{T}$ and $\vec{W}$ by moving $\vec{W}$ parallel to itself.

The vector sum $\vec{R}$ is the line segment which closes the triangle (figure 4.3.3). The usual trigonometric rules known as the law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

and the law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

apply to the triangle. Note that $\theta = 180^\circ - \alpha$ so the law of cosines becomes

$$|R| = \sqrt{|T|^2 + |W|^2 - 2|T||W| \cos(180 - \alpha)}$$

Remember, also, before you regret it, that the third side of the triangle is not the linesegment from the head of $\vec{W}$ to the head of $\vec{T}$ in figure 4.3. Another useful relation for finding the direction of $\vec{R}$ is:

$$\tan \theta = \frac{|W| \sin \alpha}{|T| + |W| \cos \alpha}$$
4.3.4 Polygon Rule (for three or more forces)

This is the general method of which the forgoing triangle method is a special case. We illustrate it in figure 4.3.4 in which the three forces to be added are \( \vec{T}, \vec{W}, \) and \( \vec{A} \). Again, keep one vector fixed (\( \vec{T} \) in this case), then move \( \vec{W} \) and \( \vec{A} \) parallel to themselves, each vector tail being placed at the head of the preceding vector. The vector which closes the polygon is the sum: \( \vec{R} = \vec{T} + \vec{W} + \vec{A} \).

The graphical solution is obtained, again, by making an accurate scale drawing, then measuring the length of \( \vec{R} \) to scale, and measuring the angle \( \theta \) with a protractor. The analytical solution is obtained by splitting the polygon into two triangles, preferably along the broken line in figure 4.3.5 and solving the the two triangles as in the triangle method.

![Polygon Method](image)

4.3.5 Component Rule (for two or more forces)

This method makes use of the resolution of a force into two components. Cartesian coordinates are used for convenience. Figure 4.3.5 shows the \( x \) and \( y \) components for the force \( \vec{W} \). The angle \( \theta \) is known and we are to find the components. If the components are denoted \( W_x \) and \( W_y \), respectively, their magnitude can be found by two methods. Graphically, the magnitudes of the components are found by completing the rectangle, with the two dotted lines drawn parallel to the axes, then measuring the length of \( W_x \) and \( W_y \).
Analytically, the two components are calculated by the trigonometric relations:

\[ W_x = |W| \cos \theta \]
\[ W_y = |W| \sin \theta \]
\[ |W| = \sqrt{W_x^2 + W_y^2} \]

and

\[ \theta = \tan^{-1} \frac{W_y}{W_x} \]

If several forces are to be added, the vector sum is most easily found by the component method. Figure 4.3.5 shows three forces on a Cartesian coordinate system, preserving the lengths of the vectors and angles. (If using the graphical method, make an accurate drawing; if using the analytical method, a sketch will suffice.)

Note that considerable labor is saved if one force is placed on the \( x \)-axis. Then obtain the \( x \)-components and the \( y \)-components of all the vectors. The magnitude of the vector sum \( \vec{R} \) is then:

\[ |\vec{R}| = \sqrt{R_x^2 + R_y^2} \]

and the direction is:

\[ \theta = \tan^{-1} \left( \frac{R_y}{R_x} \right) \]

In this example, \( R_x = A_x + B_x + C_x \) and \( R_y = A_y + B_y + C_y \).
4.3.6 Subtraction of Forces

To find the vector difference $\vec{A} - \vec{B}$, reverse the direction of the subtrahend ($\vec{B}$) and use any of the rules of adding two forces. This is obvious from the relation:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

4.4 Force Table Experiment

4.4.1 Purpose

This experiment has two purposes: To provide a laboratory experience in the addition of vectors, and to investigate whether forces follow the vector law of addition. The vectors which we will add together will represent forces, all acting upon the same object. The resultant of these vectors will represent the net force acting on the object. The object upon which these forces will act is a small metal ring to which are attached three strings. Three different forces will be applied to this ring by attaching weights to the free ends of the strings and allowing the weights to hang over the edge of the force table (using pulleys to reduce friction).

Each of the stretched strings will exert a force (pull) upon the ring. The magnitude of each force will be equal to the weight which is hanging on the other end of the string. This much is clear from intuition.

Now, you will find that once the strings are weighted and set at some arbitrarily chosen angles, the ring will tend to move. Your intuition tells you (quite correctly) that the ring tends to move in a certain direction because the three forces on the ring vectorially add up to a net force in that direction.

After some trials you will find that with certain combinations of weight values and angles, the ring will not tend to move at all under the influence of the three forces. If the forces can indeed be added vectorially, then in these cases the three forces must vectorially add up to a net force of zero. This is called an equilibrium situation; the ring, however, will sometimes not be at the exact center of the force table. Use at least 200 grams on all strings to avoid problems due to friction in the pulleys.

We can therefore use these cases to verify that our methods of vector addition apply to these forces. There are two methods for combining vectors mathematically, and both of these methods will be used:

1) We can graphically construct vectors representing the three forces on the ring and see if indeed the resultant vector is zero.
2) We can analytically add the vectors (by calculating their components), and verify that their resultant sum is zero.

It is the results of experiments of this kind that assures us that the forces on a body can indeed be represented by vectors, i.e., forces follow the laws of vector addition and the net force on a body is the vector resultant of the individual forces acting on the body.

The force table is a round table with angles marked from $0^\circ$ to $360^\circ$ around its parameter, as shown in figure 4.4.1. The ring at the center of the table is attached to three strings which pass over the pulleys at the edge of the table, and to which mass hangers and weights may be attached. The three pulleys can be moved around the parameter of the table and set to any angles desired. In the lab, we will assume that the strings are massless, and you will need to weigh your mass hangers to find how much they contribute to the force. Express the magnitude of the forces involved (and their components) in “grams” even though, strictly speaking, this is a unit of mass and not a force. Assume that the “$x$–axis” runs from the center to the $0^\circ$ mark, and the “$y$–axis” runs form the center to the $90^\circ$ mark.
**Warning**

Make sure that you include the mass of the hanger in each of the masses which you record, otherwise the results will not work out correctly.

### 4.4.2 Lab Procedure

0) Level the Force Table so that when equal masses (mass hangers + 200 grams) are placed at 120° intervals, the system is in equilibrium.

1) Using three pulleys, set up an equilibrium situation with all obtuse angles (greater than 90°). Record the angular position of each string along with the weight (you may use mass values here) attached to it. On a piece of graph paper, draw three vectors emanating from a common point representing, to scale, the three forces. Label this figure A1. On another piece of graph paper, graphically add these vectors and label this figure A2. Add the lengths of the three lines (in cm). Divide the distance of the miss (how far the sum is from the origin) by the sum of the lengths and multiply by 100 to find the percent error. Use a large enough scale that you cover most of the paper with this graph.

On the graph of figure A1, choose two perpendicular directions as $x$ and $y$ axes. Graphically find the $x$ and $y$ components of each of the three vectors. Do the $x$ components add to zero? The $y$ components?

Finally, calculate the components of the three vectors analytically, and add them:

<table>
<thead>
<tr>
<th>VECTOR</th>
<th>MAGNITUDE</th>
<th>DIRECTION</th>
<th>$x$–COMPONENT</th>
<th>$y$–COMPONENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Set up another equilibrium situation using different weights and angles, this time ensuring that one of the angles between two strings is an acute angle (less than 90°). Record the masses and force-table angles below:

<table>
<thead>
<tr>
<th>STRING</th>
<th>MASS</th>
<th>ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate the components of the three vectors, and find their sum:

<table>
<thead>
<tr>
<th>VECTOR</th>
<th>MAGNITUDE</th>
<th>DIRECTION</th>
<th>$x$–COMPONENT</th>
<th>$y$–COMPONENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td></td>
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<td>#2</td>
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<tr>
<td>#3</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Sum:</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
3) Hang the unknown mass provided from one of the strings, placing its pulley at any angle you choose. Now, using the other two pulleys and masses, set up an equilibrium situation on the force table. Record the data below:

<table>
<thead>
<tr>
<th>STRING</th>
<th>MASS</th>
<th>ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate the components of vectors #2 and #3, and find their sum. Use this information to find unknown mass #1.

<table>
<thead>
<tr>
<th>VECTOR</th>
<th>MAGNITUDE</th>
<th>DIRECTION</th>
<th>$x$–COMPONENT</th>
<th>$y$–COMPONENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#1</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Compare the mass of the unknown sample as measured on the force table, to the value measured on a triple beam balance. Determine the percentage error between these two values.

4.5 Questions

1) Throughout this experiment, we have used the units “grams” to express the magnitude of the forces acting, and yet we know grams measure mass and not force. How can we do this, i.e., how can we be sure that the laws of vector addition are being validly tested?

2) Do all of your vectors add exactly to zero? If not, how do you think that experimental uncertainty might enter into your calculation?